Polynomial integration

Name……..

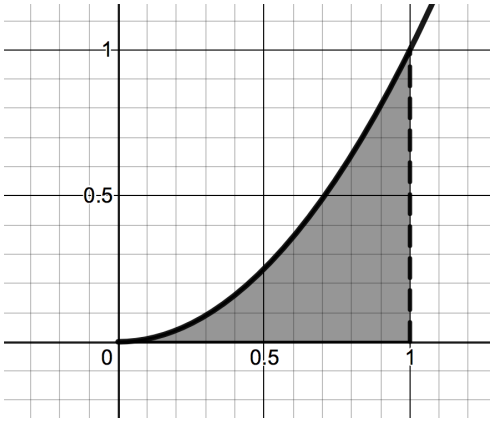
(1) Integrate the following:

(a) $\int 3x^2 + 5\,dx$ (b) $\int 0.25x^4 + \sqrt{x}\,dx$ (c) $\int x\,dx$ (d) $\int\,dx$

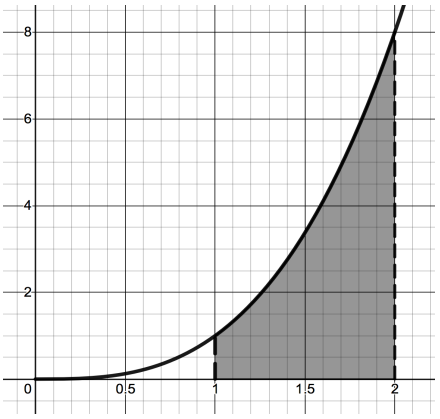
(2) Integrate the following:

(a) $\int \frac{2}{x^2}\,dx$ (b) $\int \sqrt[3]{x}\,dx$ (c) $\int 3 + 6x + 3x^2\,dx$ (d) $\int \frac{3}{5x^3}\,dx$

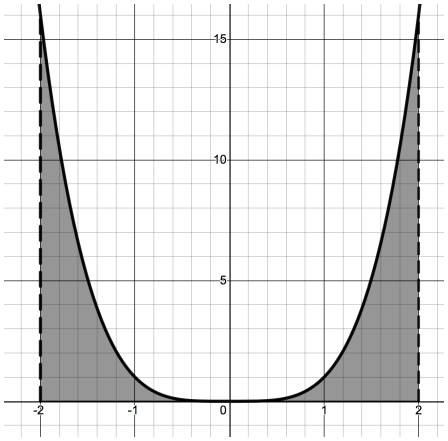
(3) Find the shaded area for the following graph:



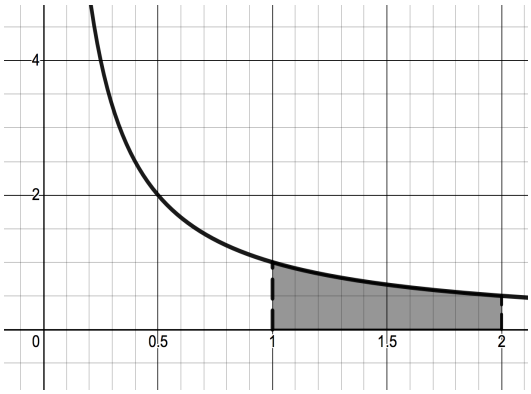
$f(x) = x^2$



$f(x) = x^3$



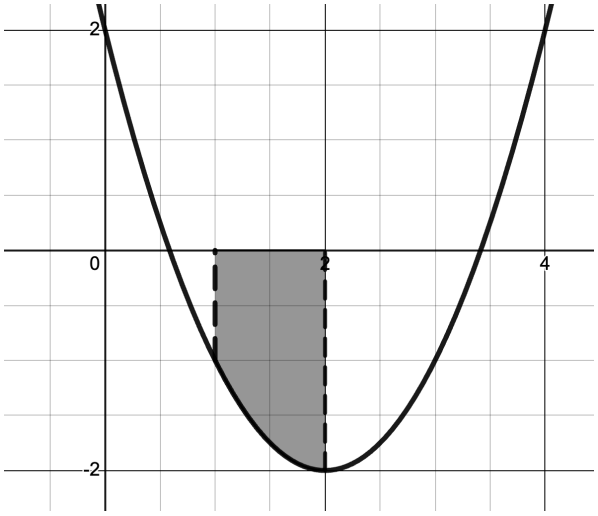
$f(x) = x^4$



$f(x) = \frac{1}{x}$

.

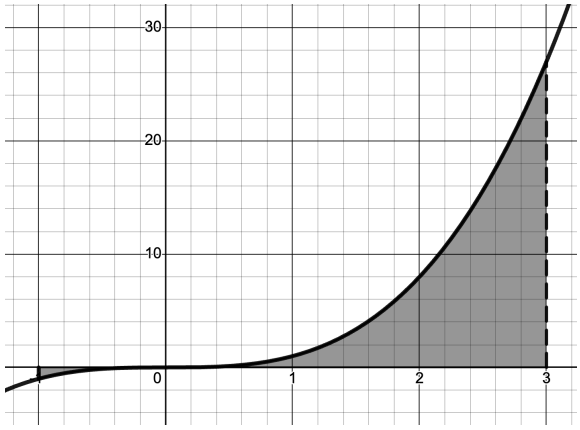
(4) The following graph $x^2 - 4x + 2$ is shaded between $x = 1$ and $x = 2$.



(a) Find $\int_1^2 x^2 - 4x + 2\, dx$

(b) Explain why your result in (a) is not the same as the shaded **area**.

(5) The following graph x^3 is shaded between $x = -1$ and $x = 3$.



(a) Find $\int_{-1}^3 x^3\, dx$

(b) Find $\int_{-1}^0 x^3\, dx$ and $\int_0^3 x^3\, dx$.

(c) Explain how to use your answer to part (b) to find the total shaded **area**.

Polynomial integration II

Name……

(1) Integrate the following:

(a) $\int_0^3 3x^2\,dx$ (b) $\int_{-2}^1 -x^2\,dx$ (c) $\int_1^2 x^3+1\,dx$ (d) $\int_1^4 \sqrt{x}\,dx$

(2) Integrate the following:

(a) $\int_1^3 \frac{2}{x}dx$ (b) $\int_0^8 \sqrt[3]{x}\,dx$ (c) $\int_1^2 x^2-x\,dx$ (d) $\int_3^5 \frac{1}{x^2}\,dx$

(3) Integrate the following:

(a) $\int_{-1}^3 (t+2)(t+3)dt$ (b) $\int_1^2 \frac{t^2+1}{t^2}\,dt$ (c) $\int_0^1 \sqrt{\frac{t^3}{4}}\,dt$

(4) For the following integrals, find a.

(a)

$$\int_1^3 ax^2\,dx = 52$$

(b)

$$\int_2^a 4x^3\,dx = 240$$

(5) A curve has gradient function defined below. If the curve passes through the point (1,10), find the equation of the curve.

$$\frac{dy}{dx} = 4x+3$$

(6) The gradient of a function is defined below. If the curve passes through the point (2,0), find the equation of the curve.

$$f'(x) = 2x^2 - x - 3$$

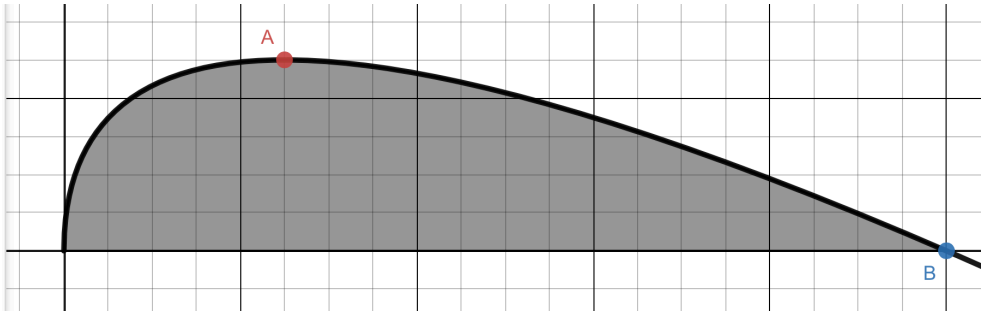
(7) The rate of growth of a population (P) of foxes in a park over time (t) in years is given by the equation:

$$\frac{dP}{dt} = 10\sqrt{t}$$

(a) If there were initially 100 foxes in the park find an equation for *P.*

(b) After how many complete years will there be more than 200 foxes?

(8) The graph of $f(x) = \sqrt{x} - x$ is shown below:



(a) Find the coordinates of A.

(b) Find the coordinates of B.

(c) Find the shaded area.

Integration rules: non polynomials

Name……

(1) Integrate the following:

(a) $\int cos\theta \, d\theta$ (b) $\int cos2\theta \, d\theta$ (c) $\int sin3\theta \, d\theta$ (d) $\int -2sin2\theta \, d\theta$

(e) $\int \sin(2\theta + 1) \, d\theta$ (f) $\int -3cos\left(\frac{\theta}{2}\right) \, d\theta$ (g) $\int \cos(4\theta - 2) \, d\theta$

(h) $\int 2\sin(\pi\theta + 2) \, d\theta$

(2) Integrate the following:

(a) $\int e^{2x} \, dx$ (b) $\int e^{5x+1} \, dx$ (c) $\int e^{0.5x} \, dx$ (d) $\int e^{-x} \, dx$

(f) $\int e^{7x+4} \, dx$

(3) Integrate the following:

(a) $\int (2 + x)^5 \, dx$ (b) $\int (4 + x)^7 \, dx$ (c) $\int (2 + 2x)^3 \, dx$

(d) $\int (5x - 2)^6 \, dx$ (e) $\int (-x + 1)^4 \, dx$ (f) $\int (0.5x + 2)^3 \, dx$

(4) Integrate the following:

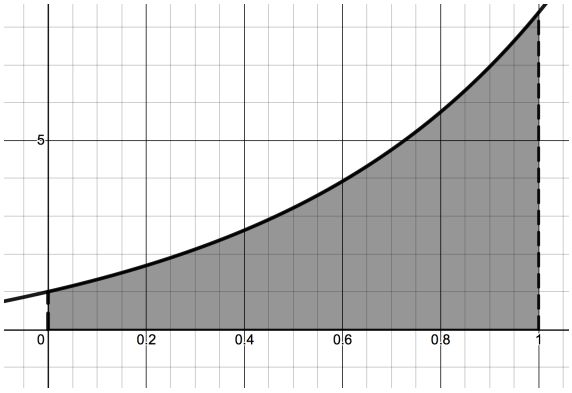
(a) $\int \frac{1}{x}\, dx$ (b) $\int \frac{1}{x+1}\, dx$ (c) $\int \frac{1}{x-5}\, dx$ (d) $\int \frac{1}{2x+5}\, dx$

(e) $\int \frac{1}{3x+4}\, dx$ (f) $\int \frac{2}{3x-1}\, dx$

(5) Integrate the following:

(a) $\int \frac{1}{(x+5)^3}\, dx$ (b) $\int \frac{1}{(2x+3)^2}\, dx$ (c) $\int \sqrt{4x+1}\, dx$ (d) $\int \frac{1}{\sqrt{2x+5}}\, dx$

(6) Below is part of the graph of $y = e^{2x} + x$



Find the area of the shaded region, leaving your answer in terms of e.

(7) Below is part of the graph of $y = \sin(2x)$

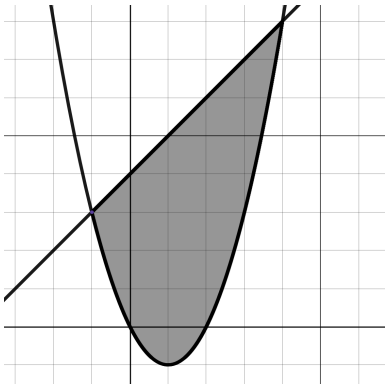


Find the area of the shaded region.

Areas between curves and Fundamental Theorem of Calculus

Name……

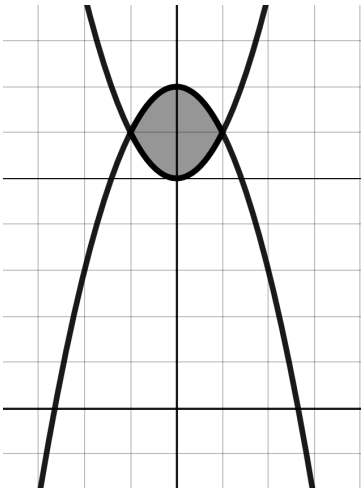
(1) The graph below shows the line $y = x + 4$ and the curve $y = x^2 - 2x$



(a) Find the x-coordinates of the intersection between the line and the curve.

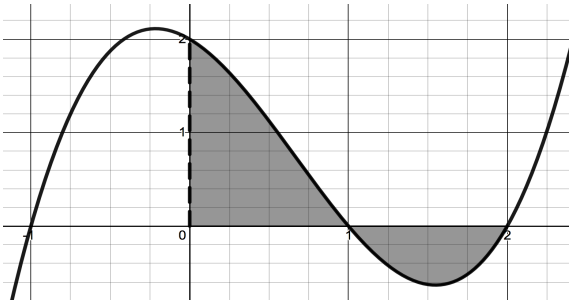
(b) Hence find the shaded area.

(2) The graph below shows the line $y = -x^2 + 7$ and the curve $y = x^2 + 5$



Find the shaded area.

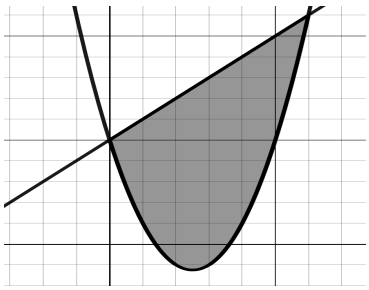
(3) A cubic graph is shown below:



(a) Show that the equation of this graph is $y = x^3 - 2x^2 - x + 2$

(b) Find the total shaded area.

(4) The graph below shows the line $y = x + 5$ and the curve $y = x^2 - 5x + 5$



Find the total shaded area

(5) Given that:

$$\int_{-1}^{3} f(x)dx = 8, \quad \int_{-1}^{5} f(x)dx = 2, \quad \int_{-1}^{5} g(x)dx = 5,$$

Find:

(a) $\int_{-1}^{5} f(x) + 2g(x)\, dx$ (b) $\int_{3}^{5} f(x)dx$ (c) $\int_{-1}^{5} f(x) + 2\, dx,$

(d) $\int_{1}^{5} f(x-2)\, dx$

Kinematics I

Name……

(1) The displacement of a particle travelling along a straight line is given by the equation:

$$s(t) = t^3 - 2t^2 + 4$$

(a) Find the particle's initial displacement.

(b) Find an equation for the particle's velocity, v.

(c) When is the particle at rest?

(ii) At what times is $v = -1$?

(d) What is the acceleration of the particle when $t = 1$?

(ii) When is the particle travelling at its minimum velocity?

(2) The displacement, from the origin O, of a particle after t seconds, travelling along a straight line is given by the equation:

$$s(t) = \sin(3t) + 2, \qquad 0 \leq t \leq 2.$$

(a) How far from O is the particle when $t = \frac{\pi}{9}$?

(b) Explain why the particle will never be at the origin O.

(c) Find an expression for the velocity of the particle.

(c) At what times does the particle change direction in the first 1 second?

(d) What is the maximum velocity the particle reaches?

(e) What is the acceleration of the particle when $t = \frac{\pi}{9}$?

(3) The velocity of a particle travelling along a straight line after t seconds is given by the equation:

$$v(t) = t^2 - 3t + 2$$

(a) Find the particle's initial velocity.

(b) Find when the particle changes direction.

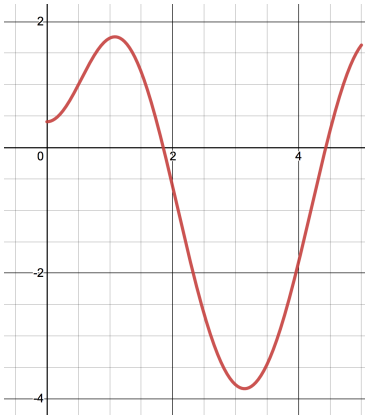
(ii) Find the minimum velocity.

(c) Find the displacement of the particle when $t = 2$

(d) Find the total distance travelled over the first 2 seconds.

(4) [GDC REQUIRED]. The graph below shows the velocity $v\ ms^{-1}$ of a particle P along a straight line after t seconds.

$$v(t) = 8\cos(t) - 5e^{\cos(t)} + 6, \qquad 0 \leq t \leq 5$$



(a) Find the initial velocity

(b) Write down the maximum **speed** of the particle.

(c) Write down the number of times the particle changes direction.

(d) Find the time when the acceleration is first 0 ms^{-2}.

(e) Find the acceleration when $t = 1$

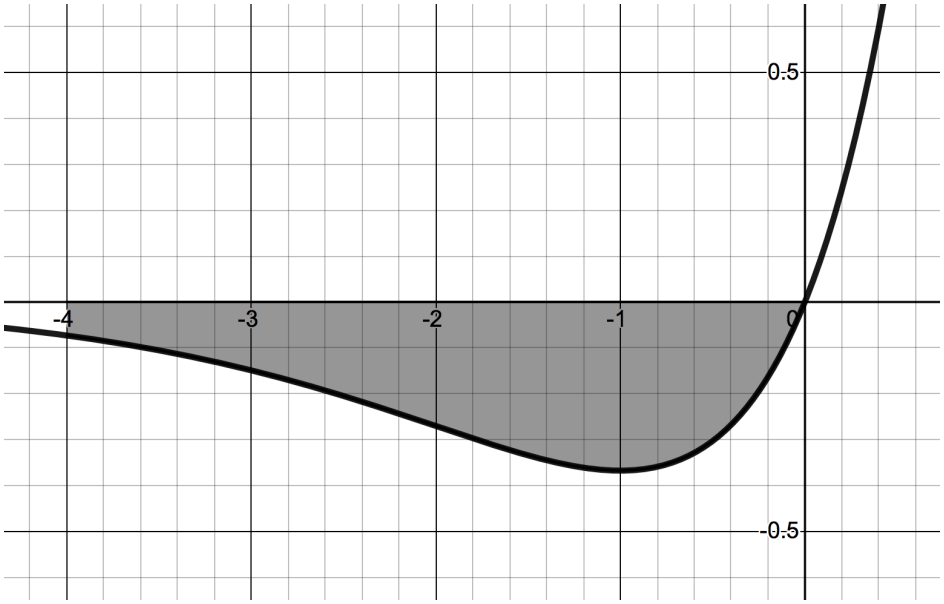
(f) Find the total distance travelled in the first 5 seconds.

Integration by parts

Name……

(1) You are given the following function:

$$f(x) = xe^x$$



Part of this graph is shown above.

(a) Find $f'(x)$.

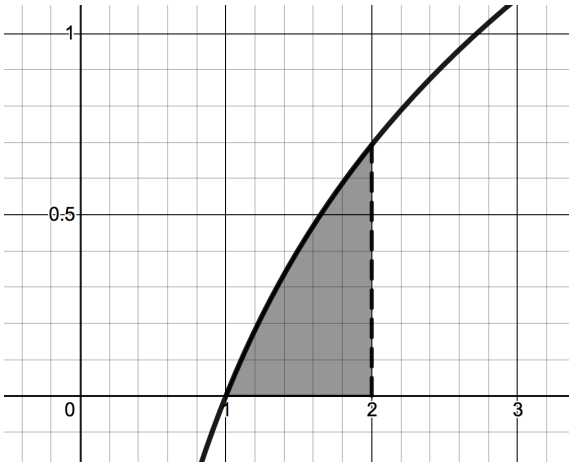
(ii) **Hence** find the exact coordinates of the minimum point. Leave your answer in terms of e.

(b) Find the exact area of the shaded region. Leave your answer in terms of e.

(c) Find a such that $\int_a^0 f(x)dx = -f'(a)$. Leave your answer in the form $pln(q)$ where $q \in \mathbb{Z}$.

(2) You are given the following function:

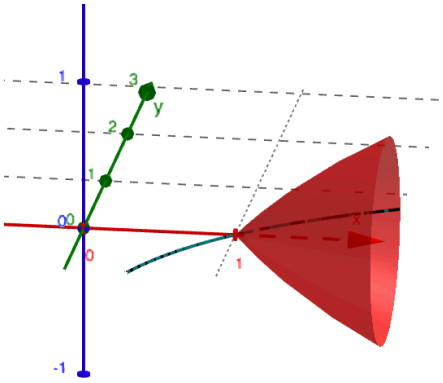
$$f(x) = \ln(x)$$



Part of this graph is shown above.

(a) Find the shaded area. Give your answer in terms of $ln2$.

(b) The area between the curve $f(x)$ and the lines $x = 1, x = 2$ is rotated 360 degrees about the x-axis.



Find the volume of the shape formed. Give your answer in terms of $ln2$.

Substitution and difficult integration HL

Name……

(1) Integrate the following:

(a) $\int \frac{1}{4+x^2}\,dx$ (b) $\int \frac{1}{1+4x^2}\,dx$ (c) $\int \frac{1}{\sqrt{1-x^2}}\,dx$ (d) $\int \frac{1}{\sqrt{4-4x^2}}\,dx$

(e) $\int sec^2(x)\,dx$ (f) $\int sec^2(2x)\,dx$ (g) $\int -csc^2(x)\,dx$

(2) Integrate the following:

(a) $\int sin^2(x)\,dx$ (b) $\int \sin(x)\cos(x)\,dx$

(3) Integrate the following:

(a) $\int \frac{2x}{\sqrt{4+x^2}}\,dx$ (b) $\int \frac{1}{x(lnx)^2}\,dx$

(4) By considering that $\frac{1}{1+e^x} = \frac{1+e^x-e^x}{1+e^x}$, find the integral of $\int \frac{1}{1+e^x}\,dx$

(5) Use the substitution $u = 2x - 1$ to find the integral $\int 2x(2x-1)^3\, dx$

(6) By using the substitution $x = a\tan\theta$ show that:

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + c$$

(7) By using the substitution $x = \sin\theta$ show that:

$$\int \sqrt{1 - x^2}dx = \int \cos^2(\theta)d\theta$$

(b) Hence find $\int_0^{0.5} \sqrt{1 - x^2}dx$

(8) By first writing $\frac{2x+1}{x^2+x-2}$ as the sum of 2 fractions with linear denominators, calculate the following:

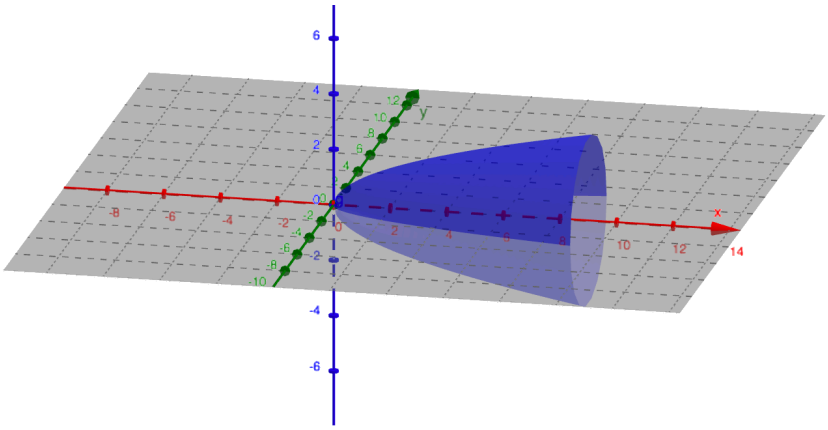
$$\int_2^3 \frac{2x + 1}{x^2 + x - 2}dx$$

Leave your answer in the form $\ln\left(\frac{a}{b}\right)$ where $a, b \in \mathbb{Z}$.

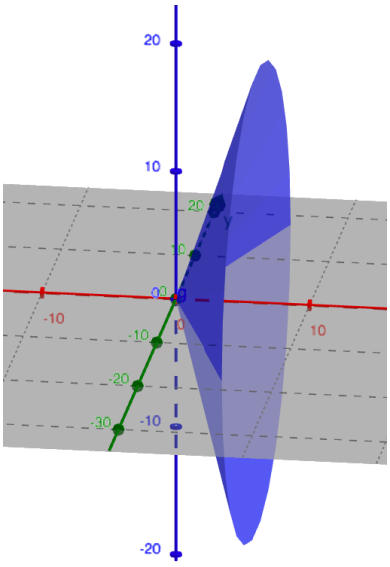
Volume of revolution

Name……

(1) Find the volume of revolution when the line $y = \sqrt{x},\ 0 \leq x \leq 9$ is rotated 360° around the x-axis.



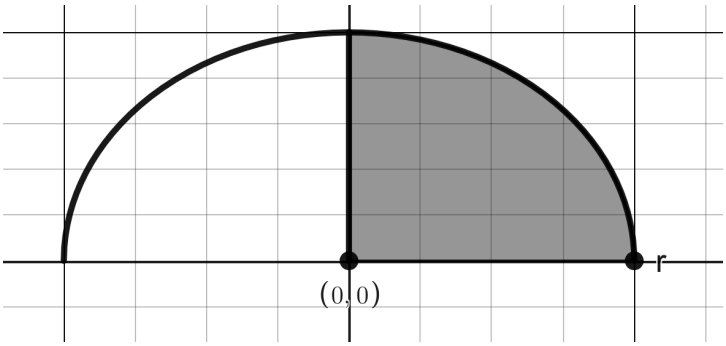
(2) Find the volume of revolution when the line $y = 3x\ \ 0 \leq x \leq 6$ is rotated 360° around the x-axis.



(b) Show that you can achieve the same result by considering the volume of the cone shown above.

(3) The general equation of a quarter-circle centred at (0,0) with radius r is:

$$y = \sqrt{r^2 - x^2} \quad 0 \leq x \leq r$$

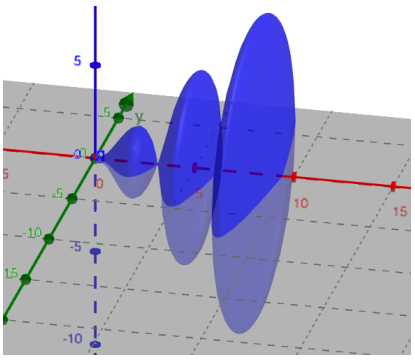


Show by use of an appropriate volume of revolution method that the volume of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

(4) [GDC REQUIRED] Find the volume of revolution when the curve shown below is rotated 360° around the x-axis.

$$y = xsin(x), \qquad 0 \leq x \leq 3\pi$$

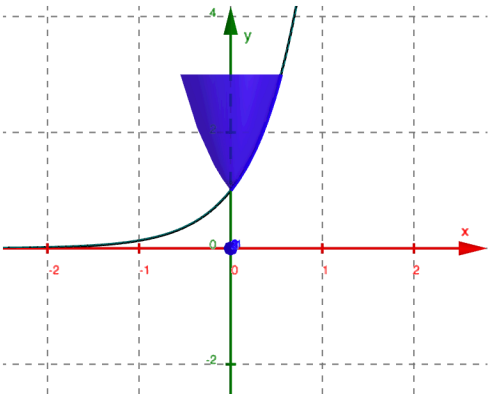


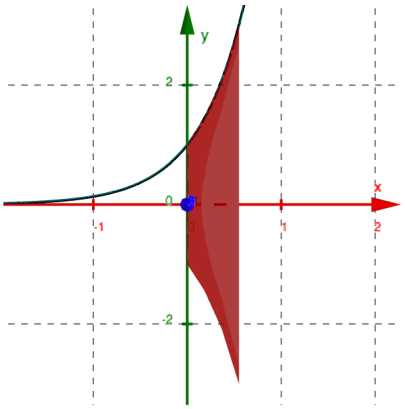
(5) The graph of $y = \sqrt{x+1}$ between $x = 0$ and $x = a$ is rotated 360° around the x-axis. The volume of the solid formed is $\frac{3\pi}{2}$. Find a.

Volume of revolution and kinematics HL [GDC]

Name……

(1) For this question we investigate volumes formed by the curve $f(x) = e^{2x}$ $1 \leq y \leq 3$.



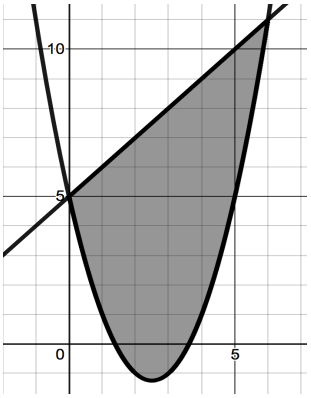


(a) A bowl is formed by rotating the curve $f(x)$ about the x-axis by 360 degrees. Find the volume of the bowl formed.

(b) A different bowl is formed by rotating the curve $f(x)$ about the y-axis by 360 degrees. What is the percentage decrease in the new bowl's volume compared with the volume in (a)?

(c) Find k so that when the curve $y = e^{2x}$ $1 \leq y \leq k$ is rotated around the y-axis the bowl has a volume of 1 unit cubed.

(2) Find the volume of revolution when the area enclosed between the line $y = x + 5$ and the curve $y = x^2 - 5x + 5$ is rotated 360 degrees about the x-axis.



(3) A particle starts at the origin and moves along a straight line so that its displacement in metres after t seconds is given by:

$$s^2 + s + t = 0$$

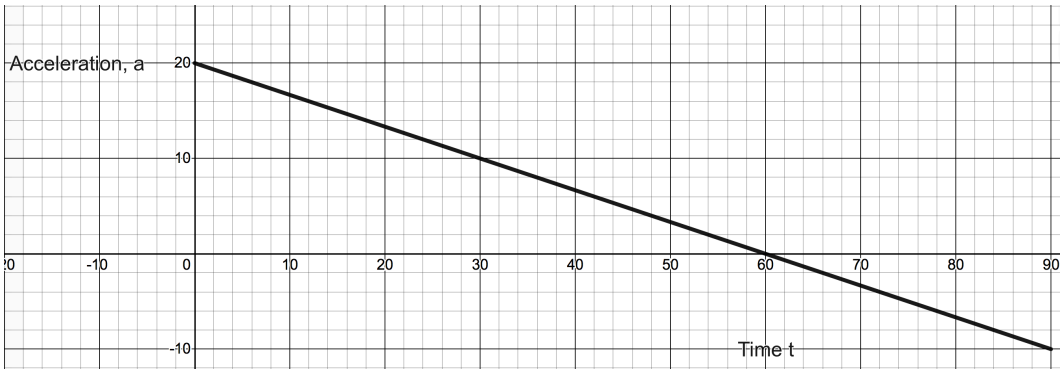
(a) Show that the velocity of the particle can be written as:

$$\frac{ds}{dt} = -\frac{1}{2s+1}$$

(b) Find the initial velocity of the particle.

(c) Find an equation for the acceleration of the particle in terms of s.

(4) Part of the acceleration $(a), ms^{-2}$ of a fighter jet travelling in a straight line over time $(t), seconds$ is shown below:



(a) Write down when the velocity of the fighter jet is a maximum. Explain your answer with reference to the graph.

(b) Find an equation for the acceleration of the fighter jet.

(c) The fighter jet is initially travelling at a velocity of 200 ms^{-1}. Find for how many seconds it is travelling faster than Mach 1 (340 ms^{-1}) for $0 \leq t \leq 120$.

(c) Find the total distance travelled by the fighter jet over the first 90 seconds.

Continuous probability density functions

Name……

(1) The continuous random variable X has probability density function that is defined as:

$$f(x) = \begin{cases} 4x^3 & 0 \leq x \leq 1 \\ 0 & elsewhere \end{cases}$$

(a) Explain why this meets the conditions of a probability density function (PDF).

(b) Write down the mode of X.

(c) Find $E(X)$

(ii) Find $Var(X)$

(iii) Find the median of X.

(d) This PDF is used to decide when someone wins on a slot machine game. If the number generator is set to pay out when $0 \leq x \leq 0.25$ find the probability that a player wins.

(e) If the slot machine costs \$1 to play and pays out \$11 when someone wins, what is the expected profit or loss for a player on this machine?

(f) In order for this to be a fair machine the operator sets the machine to pay out when $0 \leq x \leq k$. Find k.

(2) The continuous random variable X has probability density function that is defined as:

$$f(x) = \begin{cases} \frac{k}{x^2+3} & 0 \leq x \leq 2 \\ 0 & elsewhere \end{cases}$$

(a) Find k. Give your answer to 4 significant figures.

(b) Find the median of X.

(3) [GDC] The continuous random variable X has probability density function that is defined as:

$$f(x) = \begin{cases} \frac{2\cos^2(x)}{\pi} & 0 \leq x \leq \pi \\ 0 & elsewhere \end{cases}$$

(a) Find $E(X)$

(b) Find $p(X > E(X))$

(c) Given that $X > \frac{\pi}{2}$ find $p\left(X < \frac{3\pi}{4}\right)$.